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TECHNICAL NOTE

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OBSERVATIONS ON THE ROLE OF NONLINEARITY IN RANDOM VIBRATION OF STRUCTURES

By Richard H. Lyon

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OBSERVATIONS ON THE ROLE OF NONLINEARITY IN RANDOM VIBRATION OF STRUCTURES*

By Richard H. Lyon

SUMMARY

The effects of nonlinearity in several clamped-clamped beam vibration problems are reviewed. Possible effects are: jump or instability phenomena, amplitude limiting effects, and distortion of probability densities. Studies of prototype panel-frame structures show similar behavior. The necessity for extending these efforts to built-up structures is emphasized. Also, estimates for the onset on nonlinearity based on two simple models are made to show how one can be guided in experimental design by quantitative-empirical considerations.

Some theoretical techniques are appraised for their applicability to the structures problem. In particular, two approximation methods are singled out for detailed comment. Finally, the possibilities of a more creative use of experimental analysis and a closer tie between theoretical and experimental effort in structures research are explored.

*This paper was originally prepared as part of a symposium on "The Response of Nonlinear Systems to Random Excitation," which was held during Session V at the 64th Meeting of the Acoustical Society of America, Seattle, Washington, November 7-10, 1962. As presented, the title of the paper was "Empirical Evidence for Nonlinearity and Directions for Future Work." An abstract of the paper may be found in the November 1962 issue of the Journal of the Acoustical Society of America. For the present publication, the paper has been rewritten and some of the discussions have been expanded.

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FIGURE

SYMBOLS

$A(\omega)$	spectral density of acceleration
D_m	excitation power spectrum
F_p	Fourier amplitude of excitation
L	probability generating function
N_p	Fourier amplitude of response
Q	oscillator quality factor (reciprocal of loss factor g)
S	panel area
a	acceleration
c_ℓ	longitudinal velocity in the panel
$f(t)$	random force
f_1	lower frequency limit of panel response
h	panel thickness
m	dashpot parameter
p	sound pressure at the surface
s	temporal transform variable
t	time
x	transverse displacement
Φ_n	impedance of linearized oscillator
Ψ_n	displacement power spectrum

SYMBOLS (Cont'd)

α	damping coefficient
ϵ	time average membrane strain
η	nonlinear oscillator displacement
ρ_p	material density
$\rho_p h$	surface density of the panel
ϕ	response probability density
ω_o	linear oscillator resonance

I. INTRODUCTION

The problem of nonlinearity in the random vibration of structures has attracted much attention in recent years. Experimentalists have discovered many forms of anomalous behavior in random vibration testing and theoreticians have indicated that structures might be an important area for application of their theories. In this paper, we shall review some experimental studies of simple prototype structures and make observations on some theoretical methods which have been used and others which appear promising.

The field of nonlinear random vibration has a great charm, both from the theoretical and experimental points of view. The conceptual framework of probability theory, coupled with dynamical equations of nonlinear motion offers an almost unlimited scope of challenging problems to the analyst, whether his interests lie in the generation of existence and uniqueness proofs or in obtaining solutions for the response moments, probability density, stability criteria, etc. For the experimentalist also, there is a surprise around every corner. As excitation levels and spectra are changed, the response can become unstable, change its spectrum, spatial distribution, etc. One almost never quite knows what to expect.

This richness alone, of course, is reason enough for many to invest their talents in these studies. Others may be motivated by the possible applicability of their results to structures, fluid dynamics, control theory, etc. In this report, we shall concentrate on applications to structures. It is worthwhile, therefore, to try to form answers to the following questions:

1) How widespread is the problem of nonlinearity in real structures?
2) Which available theoretical techniques appear most adequate or promising for application to the structures problem?
3) What is the proper role of the experiment in studying nonlinear response of structures?

We shall not answer any of these questions here as fully as we would like. We shall find that the clues are incomplete, and a fair degree of judgment and educated (?) guess will be involved in the partial answers we give. The paper will therefore have a stronger editorial flavor than some would prefer. We hope it will stimulate many readers to turn detective and uncover additional evidence on the nature of the crime.

Our plan for the paper is as follows. We begin by showing the variety of ways nonlinearity can manifest itself experimentally in a very simple structure, the clamped beam.

We do this by reviewing several experiments on this system. We then review some studies of prototype structures with particular emphasis on the implications the results may have for real structures. Then we indicate how estimates may be made of the onset of nonlinear behavior in structures when one has a general idea of the form nonlinearity may take. Then, we review some theoretical results and methods which appear promising, and finally, we suggest that a greater role of the experiment in the analysis of nonlinear behavior is possible.

II. EFFECTS OF NONLINEARITY IN LABORATORY STUDIES

Nonlinearity as an experimental quality may be broadly described as a lack of proportionality between excitation and response. This may manifest itself in many ways since the response has many descriptions and various parts of the description may show nonlinarity to varying degrees. In this section we show with experimental records how nonlinearity affects rms response, stability, and probability densities.

A system which has been widely studied is the clamped-clamped beam with axial restraint. The transverse deflection with axial restraint causes a membrane strain in the beam which adds to the restoring bending strain, resulting in a hardening type of stiffness nonlinearity in the displacement response. Since the membrane strain is an even function of the displacement, it causes a different form of nonlinearity in the measured strain at any point. Displacement statistics and strain statistics for this system are therefore not the same, but our purpose here is merely to show the effects of nonlinearity, and for this, either will suffice.

In his studies of the first mode strain response of a beam, P. W. Smith, Jr. (ref. 1) used the configuration shown in Figure 1. The beam was mounted in the side wall of an acoustic duct and excited by an intense monochromatic sound wave. In Figure 2 we show the results of this experiment. One notes the typical nonlinear response curve characteristic of a hard spring behavior - the "jumps" in amplitude tracing out a hysteresis type of curve, and a broadening of the loop as the excitation amplitude is increased. Finally, in Figure 3, the strain at resonance versus SPL shows the amplitude limiting effects of the stiffening system.

A system very much like this was studied by Heckl (ref. 2), but his excitation was narrow band noise rather than a pure tone. A diagram of his beam is shown in Figure 4. It was excited mechanically with an attached coil and the vibration sensed with an accelerometer. The low amplitude first mode resonance was at 75 cps with a quality factor (Q) of 7. The excitation was filtered noise of 8% bandwidth, slowly swept through the resonance region of the first mode. The low level response is shown in Figure 5a, indicating a general rise in response near resonance. As the excitation is increased, however, we see in Figure 5b a tendency for the rms level to become unstable in the 76-82 cps range, the oscillator apparently making transitions between two metastable rms levels of response. This could be heard clearly. Unlike the pure tone response, however, there was no evidence that the pattern of response differed depending on the past history of excitation. Apparently for noise response, an "equilibrium instability" is achieved, the pattern changing uniformly as the enter frequency of excitation is changed.

Finally, an alteration in the probability distribution of strain with increased excitation has been studied by Smith, Smits, and Lambert (ref. 3). A diagram of their bar is shown in Figure 6. In order to eliminate damping at the clamps, the bar and its supports were cut out of a single block of aluminum. The strain at the center of the beam was measured and processed to yield probability densities for the maxima and minima of total strain, including both bending and membrane contributions. The distribution of maxima is shown in Figure 7. We note that it departs from the theoretical Rayleigh distribution, having a higher probability of larger maxima. This is due to the in-phase addition of the membrane and bending strains. The distribution of strain minima is shown in Figure 8. Theoretically there is a limiting negative strain, and the experimental data tend to support this. The nonlinearity is clearly evident through the non-Rayleigh form of the distribution and the lack of symmetry with the distribution of maxima.

In addition to the nonlinear effects described, there are others of equal importance. A change in spatial distribution of motion was noted by Smith (ref. 1), and changes in the frequency spectrum of response must also occur. It would appear, therefore, that there are several tests which can be applied to detect nonlinearity. Unfortunately, when one tries to apply them to real structures in a field situation, difficulties may arise in the interpretations. In the following section we shall indicate how some prototype structures have been examined for nonlinear behavior.

III. NONLINEAR BEHAVIOR OF PROTOTYPE STRUCTURES

As we have seen, it is not very difficult to obtain nonlinear behavior in dynamical systems. It is common knowledge among test and environmental engineers that structures and components can display unexpected behavior in a test environment, and frequently this behavior is described as arising from "nonlinearity." There are also simple examples of real life random vibrations which have obvious nonlinearities. An important class of these concerns vibration limiting effects, such as mount bottoming, relay chatter, window rattling, etc. Experience with this type of nonlinear behavior is so common that it is perhaps not necessary to document it. Since it is readily detected and normally represents a serious malfunction of the system, design efforts are aimed at preventing such behavior. The dynamical nonlinearities of panel-frame structures are not so dramatic in their effects, and for this reason several investigators have tested prototype structures for more detailed studies of response behavior.

An example of this is the nonlinear behavior of flat and curved panels studied by Lassiter, Hess, and Hubbard (ref. 4). These panels, which are shown in Figure 9, were fastened at their edges in such a way as to inhibit in-plane motion and exposed to siren and turbojet noise. The pure tone response (primarily first mode) is shown in Figure 10 for the flat panel and again it shows the characteristic "hard-spring" behavior. In Figure 11, we note that the curved panel shows "soft-spring" behavior, probably something like an "oil canning" effect. The strain response of the flat panel to jet noise is shown in Figure 12. The response limiting effect of the membrane stresses is evident. There seems little doubt that these panels are exhibiting non-linear response under excitation amplitudes of engineering interest.

If one may presume that the work we have been describing has been carried out in order to understand how real structures vibrate, then it is appropriate to inquire whether similar panels mounted in real aircraft structures would display a similar sort of nonlinearity. One can only decide this on the basis of field data for tests of real structures. If one finds that in fact real structures display no nonlinearity, then the goal of the laboratory setup should be to test the panels in such a way that they vibrate linearly; cr if the field test displays nonlinearity, then the test should seek to reproduce the same form of nonlinearity.

It becomes crucial, therefore, to sort out types of non-linear behavior in field data. This is by no means an easy task. If we consider the ways in which nonlinearity was detected in Section II, we find that field conditions make them more difficult to apply. One possibility is to measure the membrane stresses in the panel, since these indicate whether a stiffness nonlinearity may occur. Such a measurement has been made by Freynik (ref. 5) for glass panels excited by noise from a blowdown tunnel. Freynik's test window is shown in Figure 13. It was mounted in its frame with putty and in a test cubicle to shield one side from the noise. In a third octave band centered at the fundamental resonance of the panel-cavity combination, a 20 db increase in acoustic excitation resulted in an increase in only 9.1 db of bending strain and a 20 db increase in membrane strain (the apparent proportionality between sound pressure and membrane strain is presumably fortuitous). The response-excitation relation is shown in Figure 14. Again, the amplitude limiting effects of membrane strain are apparent.

IV. ESTIMATING THE ONSET OF NONLINEAR BEHAVIOR

There is ample evidence that nonlinearity can be significant in the vibration of laboratory and prototype structures. Unfortunately, there is almost a complete absence of evidence for nonlinear response of built-up structures in either natural or artificial environments. There should be measurements made of such structures, and perhaps the new sonic fatigue facilities at Langley Research Center and the Aeronautical Systems Division will provide data along these directions. In the meantime, one can make estimates of the levels required to produce significant amounts of nonlinearity based on conceptual models of the causes of the nonlinearity. In this section, we consider the consequences of two models of nonlinearity; membrane stress effects and slip damping. Both types of nonlinearity have been discussed as potentially significant in panel response.

Of course, there is a sense in which there is no problem in deciding whether a panel vibrates nonlinearly, since in detail <u>all</u> panels are nonlinear. The key point for the environmental or structures engineer, however, is whether or not linear theory can estimate response levels. In order for nonlinearity to be significant, the alterations in response which it produces must rival in amplitude the uncertainty in the linear predictions which under the present state of the art, may be considerable.

The vibration of a flat plate with fixed edges produces a time average membrane strain ϵ given by

$$\epsilon = \overline{|\nabla x|^2}/2S \ , \tag{4.1}$$

where S is the panel area and x is the transverse displacement. In order to estimate ϵ, we note the empirical result that the skin vibration of aircraft and spacecraft boosters is approximately 20 db above mass law (ref. 6), i.e.

$$\overline{a^2}/\overline{p^2} = 100/\rho_p^2 h^2 \tag{4.2}$$

where a is the acceleration, p is the sound pressure at the surface, and $\rho_p h$ is the surface density of the panel. This response is generally fairly uniform in the first three octave bands above 200 cps.

Assuming a uniform reverberant vibrational field with this spectrum, we compute the required sound pressure to produce a membrane restoring force equal to the bending forces. The result of this calculation is

$$\overline{p^2} \approx 16\rho_p^2 h^4 f_1{}^4 = 163 \text{ db re } .0002 \text{ dynes/cm}^2, \tag{4.3}$$

when f_1 (lower frequency cutoff) is 200 cps, $\rho_p = 2.7$ gm/cm^3 (aluminum) and h = 0.25 cm (typical value of spacecraft skin thickness). Typical overall sound levels for a large current missile in the tank areas is 140 db, and about 155 db in the engine compartment. We would very likely guess that membrane stiffness nonlinearity is not a serious factor in its response. If we apply these estimates to the aircraft skin, which is normally more of the order of 0.08 cm in thickness, then the estimate becomes 143 db. This is again higher than the acoustic levels normally present on the surface of aircraft by about 10 db.

In composite structures of panels and frames held together by rivets, the measured damping is usually higher than that which one would get from hysteretic material damping alone. It has been postulated, therefore, by Ungar (ref. 7) and Mead (ref. 8), among others, that the damping is associated with contact surfaces near the rivets. Mead (ref. 8) has measured the damping of a riveted joint and reported the result shown in Figure 15. There is clearly a transition to a nonlinear damping at about one poundal of rivet loading.

One does not know, at this point, the cause of this transition but we may expect it has something to do with the relative magnitude of the surface motions and the size of the surface irregularities. For the surfaces usually encountered, the irregularities (asperities) are of the order of one micron. The surface motions are estimated from the mean square bending strain $\overline{\epsilon_b^2}$ at the panel surface, which is

$$\overline{\epsilon_b^2} = \frac{3}{c_\ell^2} \int \frac{d\omega}{\omega^2} A(\omega) \tag{4.4}$$

where c_ℓ is the longitudinal velocity in the panel and $A(\omega)$ is the spectral density of the mean square acceleration of a reverberant vibrational field.

We again assume a constant acceleration over 3 octave bands above 200 cps, with vibration levels 20 db above mass law. The mean square bending strain then is

$$\overline{\epsilon_b^2} \approx 2\overline{p^2}/f_1^2 c_\ell^2 \rho_p h^2 \qquad (4.5)$$

For an SPL of 126 db, the rms bending strain of an .032" aluminum panel is 4×10^{-4}. If the dimension of the joint is of the order of centimeters, then surface displacements of the order of microns will be achieved at this SPL. Based on our rather uncertain hypothesis that this is the source of non-linearity, damping nonlinearity in aircraft would appear to have an onset some 20 db below the stiffness nonlinearity.

From the estimates concerning these particular structures, we might conclude that while damping nonlinearity is a possibility, the chances of stiffness nonlinearity are more remote. The point, however, is not to draw precise conclusions from our estimates but to indicate how estimates can be made to tuide us in deciding which forms of nonlinearity may occur at acoustic levels of interest. Subsequently, experiments should be designed to enhance the particular aspect of non-linear behavior to be examined.

V. THEORETICAL TECHNIQUES APPLICABLE TO STRUCTURES PROBLEM

The requirements for a good theory of nonlinear structural response to noise are the same as those for a good linear theory. Using it, one should be able to predict the frequency and spatial distributions of strains, accelerations, deflections, etc. to an accuracy compatible with the uses to be made of the information, and the amount of effort available for the calculations.

In addition, it may be necessary for the theory to predict probability densities and other higher order statistics of the response. It should do all these things with the minimum use of empiricism on one hand and intricate and detailed computations on the other. It must contain as a part of its central structure a recognition of the complexity and multiple mode behavior of real structures. It should, in short, be eloquent in its simplicity and fecund in its interpretations.

The major theoretical emphasis until the present has been on the two degree of freedom oscillator which describes the motion of one mode of a linear structure. [We note here the unfortunate engineering usage of the term "single degree of freedom" system for a mode of vibration. This nomenclature should be avoided.] There is no need to recount here in detail the theoretical achievements except to note that some first order probability densities and moments of the response have been computed. One of the important unsolved problems which remain is the calculation of the spectra of response, which is related to the solution of the nonstationary statistics of the oscillator.

The usefulness of a single mode model in nonlinear vibrations probably depends on the mode distribution in frequency space and the nonlinear mechanism. A mass sitting on a (nonlinear) mount that hardens, or perhaps bottoms, does not have its dynamical description changed by the nonlinear action. A beam, on the other hand, may have a predominant mode which "becomes nonlinear" as the amplitude of response increases. One may find, as did Smith (ref. 1), that a combination of nonlinear damping and resistance is necessary to explain the observed response. Smith also observed, however, that the mode shape appeared to change as amplitude was increased (ref. 1). Under these circumstances the meaning of a mode of vibration becomes rather hazy.

For structural problems, a single mode description is probably inadequate, and there have been only very limited results in multimode analysis. Ariaratnam (ref. 9) has been able to compute the joint probability density of velocity and displacement for two stiffness coupled oscillators with nonlinearity in the stiffness, excited by white noise generators. He was able to get solutions for ratios of excitation spectral density to damping corresponding to thermal equilibrium, and his probability density is equivalent to the Boltzmann distribution (ref. 10). It is possible to obtain these densities for any number of modes with an arbitrary amount of stiffness nonlinearity, if the ratio of noise spectral density to damping for each mode is the same, which is analagous to thermal equilibrium. No exact solutions have been found for unequal ratios nor has anyone found exact expressions for the densities when resistance and mass elements are nonlinear.

If we could postulate a panel-frame structure with stiffness nonlinearity only, then the "equilibrium" or Boltzmann density would result in an acceleration spectrum for the panel in octave bands rising at 9 db/octave. This is because the number of structural modes doubles in each octave band and each mode has the same energy and hence the same velocity. This kind of acceleration spectrum is not usually observed for these structures and we conclude that the "thermal equilibrium" spectrum is not characteristic of real structures.

Considering then the restrictions on the usefulness of available solutions and the very limited number of available exact solutions of the Fokker-Planck equation, it would appear that there is little hope of progress in this direction. The development of approximate techniques in two different areas suggests theoretical approaches which may be fruitful. One of these is a study due to Daniels of the use of saddlepoint methods for the approximate solution of certain difference-differential equations to which the Fokker-Planck equation is closely related (refs. 11 and 12). The second development which deals directly with moments of the nonlinear equations of motion has been put forward by Kraichnan (refs. 13 and 14).

In this context, we can only outline these approaches, but that is perhaps sufficient to indicate why we think they are useful. The processes studied by Daniels are univariate, so we cannot apply them to the two degree of freedom oscillator. Consider, however, the Fokker-Planck equation

$$\frac{1}{2}\frac{\partial^2\phi}{\partial x^2} + \frac{\partial}{\partial x}\, m(x)\,\phi = \frac{\partial\phi}{\partial t} \tag{5.1}$$

which would govern the response density ϕ of a mass-nonlinear dashpot system under white noise excitation. The system is known to have the configuration $x = \xi$ at $t = 0$. If the Laplace transform of ϕ is $P(x,s)$, then defining $L = \ell nP$, one can form a nonlinear differential equation for L. In the spirit of the WKBJ (ref. 15) method, Daniels assumes that $\partial^2L/\partial x^2$ may be neglected compared to $\partial L/\partial x$ and its powers. Solving for $\partial L/\partial x$, and integrating to find the solution, one has

$$L^{\pm} = \int_{\xi}^{x} \left\{ m(w) \pm \sqrt{m^2(w) + 2s} \right\} dw \tag{5.2}$$

The two branches must be chosen for proper behavior of the density for the separate regions $x \gtrless \xi$. The Laplace transform of ϕ is thus reduced to quadratures. The inversion integral is therefore

$$\phi(x,t) = \frac{1}{2\pi i} \int \exp[L^{\pm} + st]\, ds. \tag{5.3}$$

Daniels suggests that this may be solved by the method of steepest descents (ref. 16). The saddlepoint $s_o(x,t|\xi)$ is given by

$$t = -\frac{\partial L^{\pm}}{\partial s_o} = \mp \int_{\xi}^{x} \frac{dw}{\sqrt{m^2(w)+2s_o}} , \tag{5.4}$$

and the formal saddlepoint solution to (5.3) is

$$\phi(x,t) = \left[2\pi \frac{\partial^2 L^{\pm}}{\partial s_o^2}\right]^{-1/2} \exp[L^{\pm}(s_o) + ts_o] . \tag{5.5}$$

It would be idle, of course, to pretend that this formal answer is sufficient. One must proceed to find the form of s_o and to verify that the approximations are reasonable. In addition, the method should be extended to more variables. Nevertheless, solutions are possible with this approach which have not been achieved in other ways. It warrants further study.

Kraichnan's "method of stochastic models" has been applied with success to the problem of isotropic turbulence (in that he was able to get a solution for the energy spectrum). Basically his approach is as follows. He writes the coupled nonlinear equations of motion for the degrees of freedom of the system.

For a turbulence field these are the spatial Fourier amplitudes, for a nonlinear oscillator they are the spectral amplitudes in frequency space. The nonlinear interaction terms are replaced by statistical interactions of the mode with an infinite set of other modes. The statistics of the interaction terms are evaluated by making certain assumptions regarding sources of coherence in the interacting modes (ref. 14).

By following this through, one finds that the "zero'th order approximation" is just the method of equivalent linearization. This is reassuring, since it is not only a well-known method, but it is also a very useful one. Moreover, Kraichnan's method tells one how to go on beyond equivalent linearization - it is not a small step, but the procedures are defined. Following this through, for example, for the nonlinear oscillator should produce results for the spectrum changes due to nonlinearity beyond the mere shift in resonance frequency which one gets from equivalent linearization.

Consider the nonlinear oscillator with a hardening spring

$$\ddot{\eta} = 2\alpha\dot{\eta} + \omega_o^2 (\eta + \eta^3) = f(t) \qquad (5.6)$$

where f(t) is a random noise source of known spectral density. The response and excitation are expanded into their complex

Fourier amplitudes N_n and F_n over the interval $(-T,T)$. Equation (5.6) then becomes the nonlinear algebraic relation

$$\Phi_n N_n + \omega_o^2 \sum_{p+q+r=n} N_p N_q N_r = F_n \tag{5.7}$$

where

$$\Phi_n \equiv \omega_o^2 - 2i\alpha\omega_n - \omega_n^2 . \tag{5.8}$$

The mean square of η is

$$\overline{\eta^2} = 2 \sum_1^{\infty} \overline{|N_m|^2} \equiv 2 \sum_1^{\infty} \Psi_m \tag{5.9}$$

and thus Ψ_m and

$$D_m \equiv \overline{|F_m|^2} \tag{5.10}$$

are the power spectra of η and f respectively.

We solve (5.7) for the power spectrum of η by multiplying with N_n^* and averaging. The result is

$$\Phi_n \Psi_n + \omega_o^2 \sum_{p+q+r=n} \overline{N_p N_q N_r N_n^*} = \overline{F_n N_n^*} \tag{5.11}$$

It is at this point that one applies Kraichnan's approximations in evaluating the triple sum. The simplest assumption or "zero'th order approximation" is that the n^{th} amplitude is interacting with a completely independent set of amplitudes so the only possibility for nonzero averages occurs whenever

$$p = -q, \; r = n$$

or, $$p = -r, \; q = n$$

or, $$p = n, \; q = -r.$$

Under these circumstances, the solution of (5.11) is

$$\Psi_n = \frac{\overline{F_n N_n^*}}{\Phi_n + 3\omega_o^2 \overline{\eta^2}} \,, \tag{5.12}$$

which is the spectral form of the equivalent linearization result (ref. 17).

In seeking a higher approximation, it is necessary to look at triad interactions $N_\alpha N_\beta N$ such that $\alpha + \beta + \gamma = n$. These interactions are assumed to behave like perturbations in the excitation of the n^{th} mode. They therefore begin to affect the excitation as illustrated by Eq. (5.12). The response function is also altered, however, by the interaction. We shall not spell out the development of the relations; they lead to complicated nonlinear algebraic equations for the response spectrum. As in the case of Daniels method, we do not find complete answers available, but the methods suggest the usefulness of further development.

VI. THE USES OF THE EXPERIMENT IN STRUCTURAL ANALYSIS

The discussion of the previous section has centered on theoretical methods of structural analysis. It is doubtful, however, that any purely theoretical approach will generate engineering answers to the problems of structural response. The classes of structures are diverse, they contain very many degrees of freedom, and the methods of construction are such that the problem in a real sense defies description. The problem is even more fundamental, however, because even if one could define the structure through its dynamical equations and boundary conditions, the answers would be so very complex that no one could take the time or muster the interest to read them.

In viewing such a situation, paradoxically, the acoustician takes heart, for he is familiar with the successes of room acoustics, including impact noise and transmission of sound through structures which are systems every bit as complicated and ill defined as the aerospace structures we have been discussing. The success of room acoustics has come from the idea of describing the dynamics in a statistical way and being willing to accept answers which are averages of the simultaneous effects of very many modes. In addition, there has been a willingness from the beginning to use experimental analysis in a creative way to sort out the really significant parameters in the response, to test simplifying assumptions about the state of the motion, and to check in a practical way the utility of the simple theories which have been generated in this process.

This use of experimental analysis as a partner in the development of a theory rather than a simple check on one's ability to model his equations with a dynamical system is very much needed in present day structural analysis. There has been an unfortunate tendency in some contemporary centers of engineering science to make these separate functions. It is unfortunate because the very nature of the structures problem ideally would require the same man to do both the theoretical and the experimental analysis. At the very least there should be mutually beneficial closely integrated effort between the theoretician and the experimentalist.

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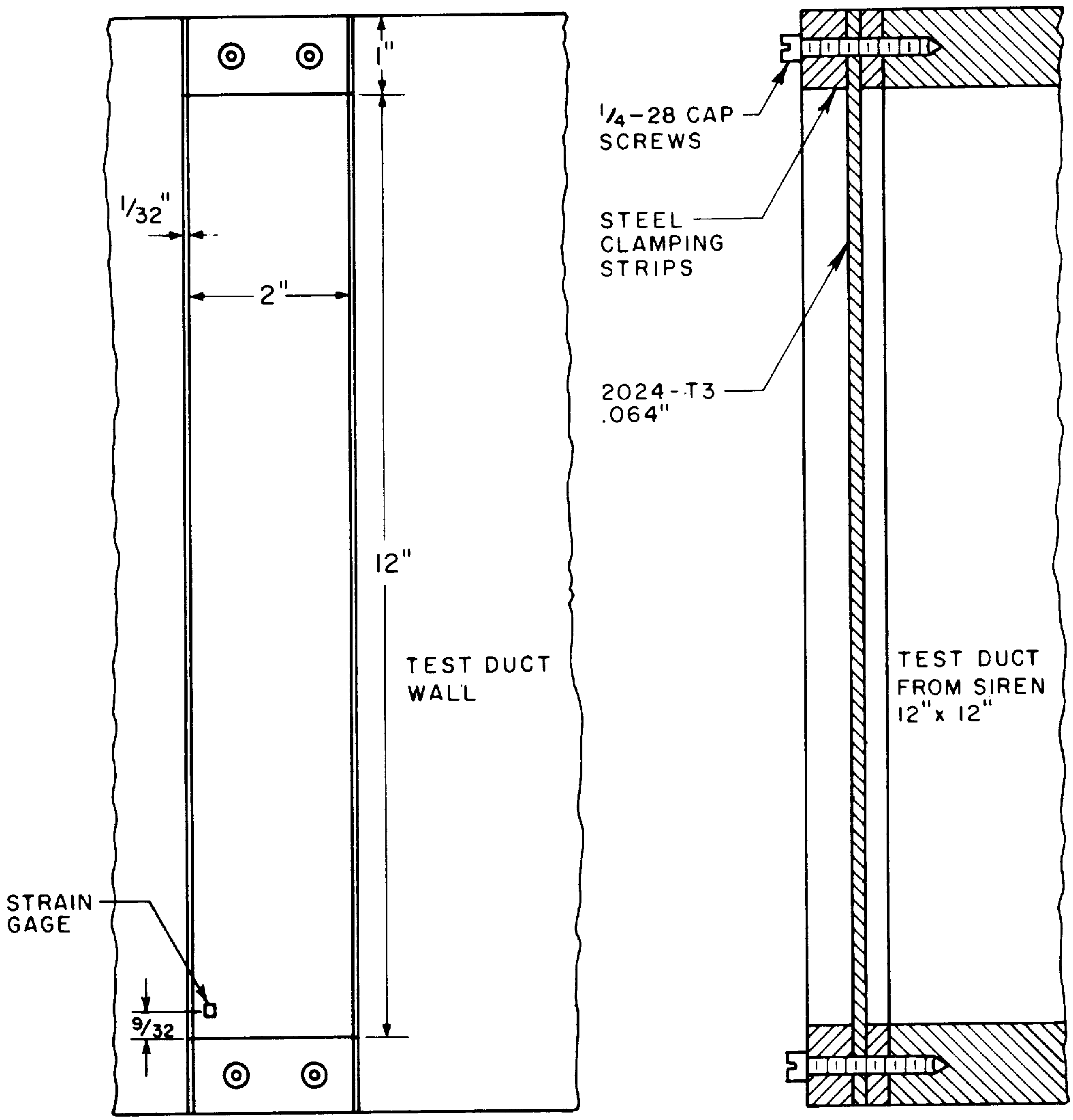


FIG. 1 DIMENSIONS AND MOUNTING DETAILS FOR SMITH'S EXPERIMENTAL PANEL (REF. 1)

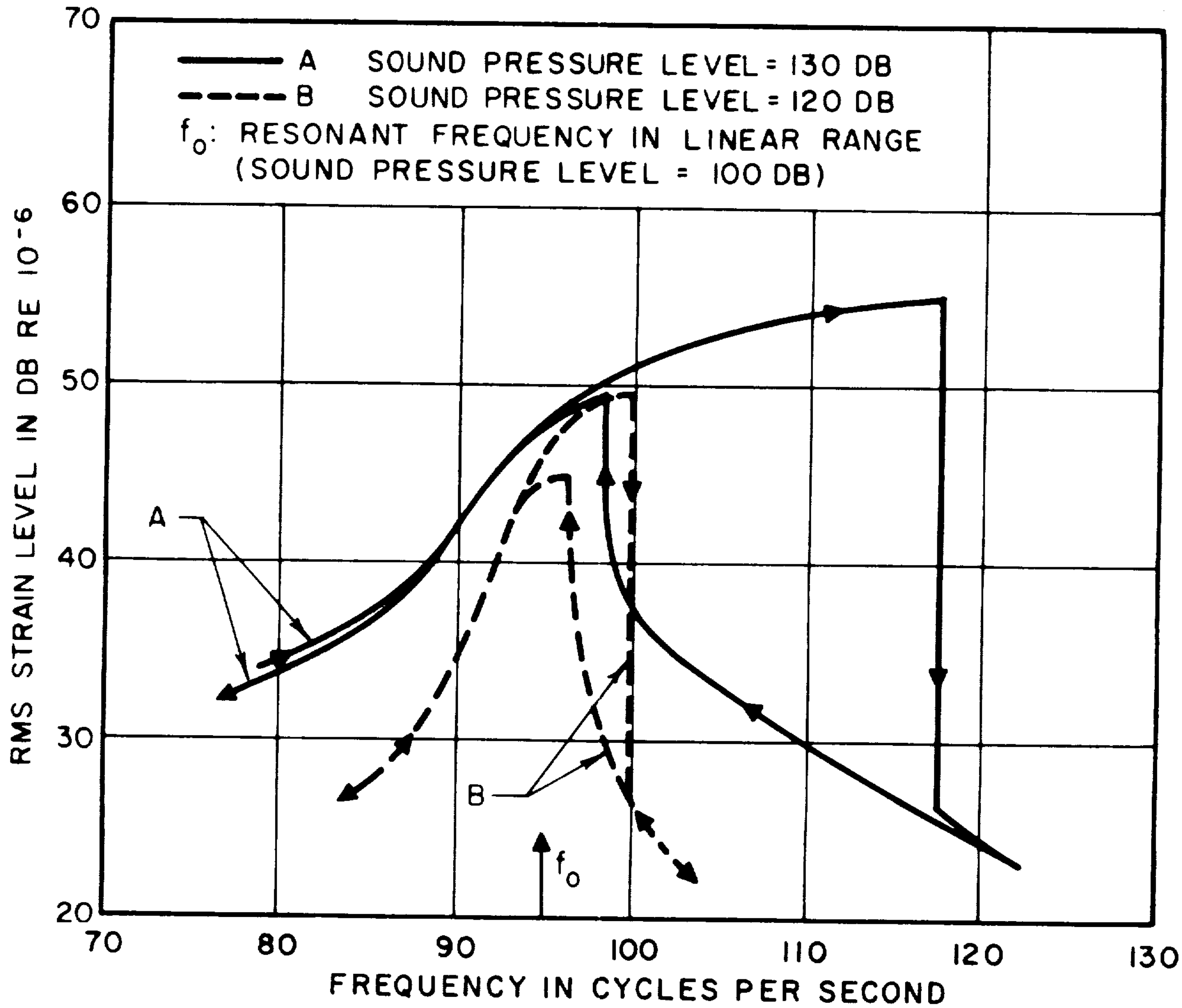


FIG. 2 RESPONSE CURVES FOR SOUND PRESSURE LEVELS OF 120 AND 130 DB AT THE FACE OF PANEL (REF. 1)

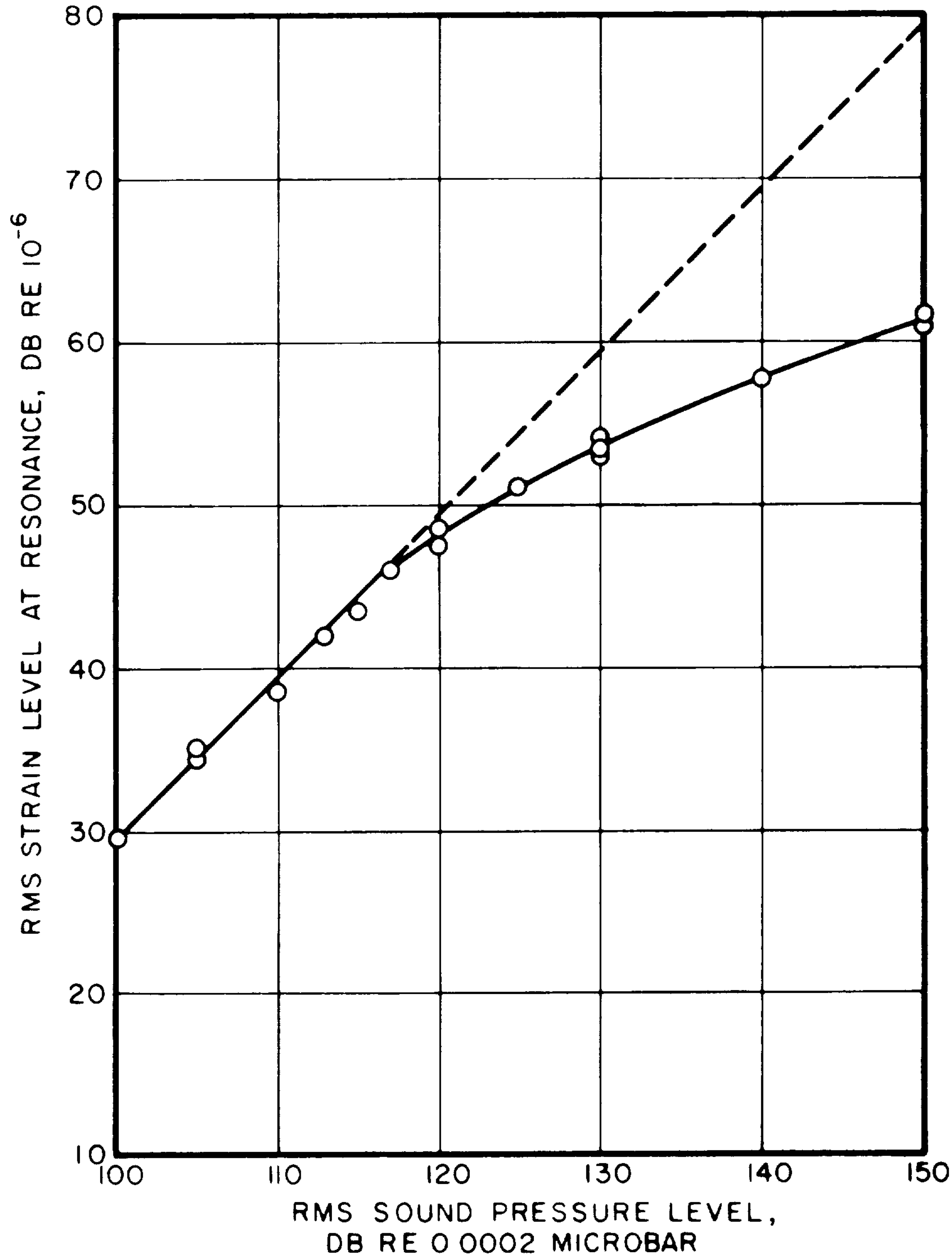


FIG. 3 RESPONSE STRAIN AT FUNDAMENTAL RESONANCE VS. SOUND PRESSURE LEVEL (REF. I)

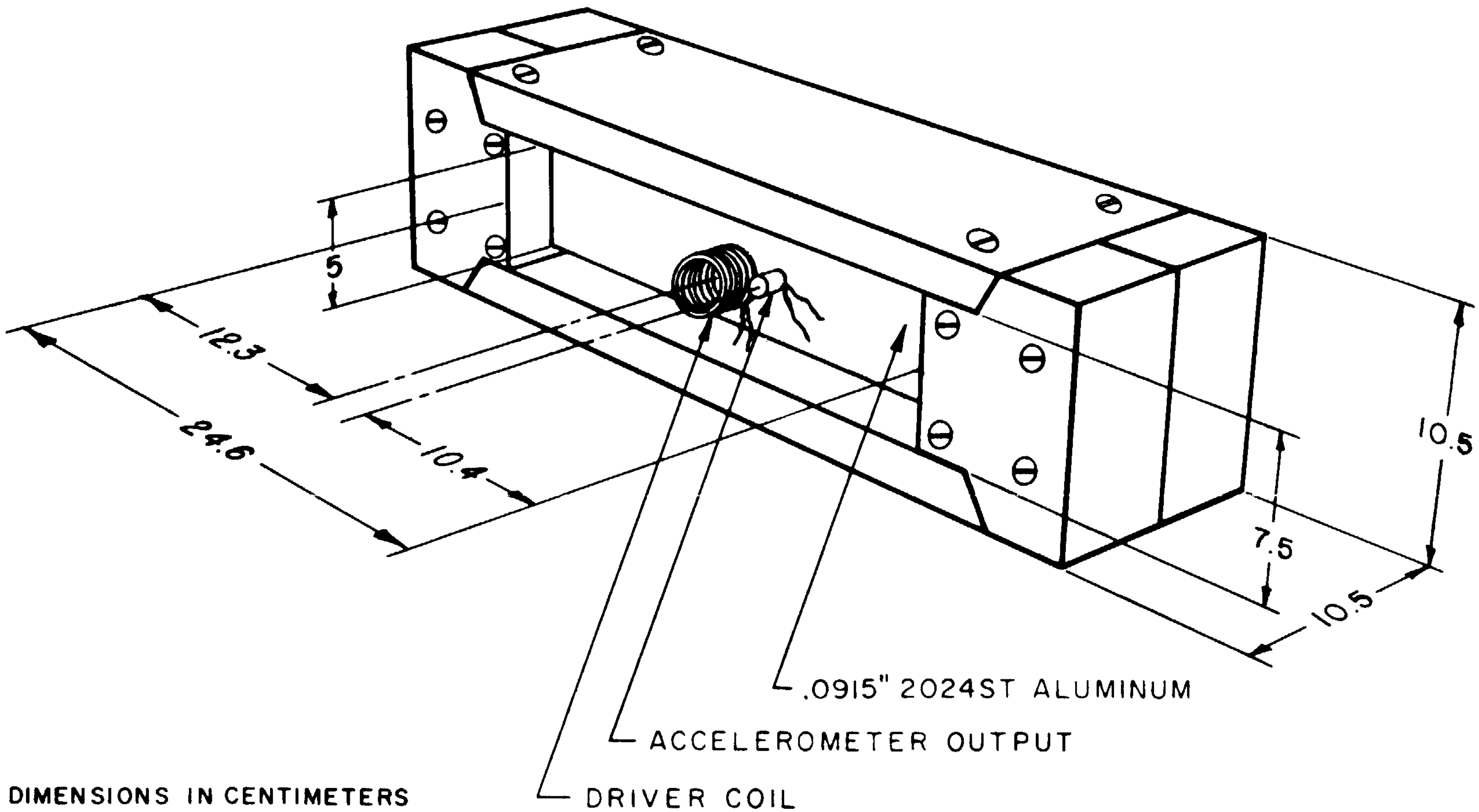
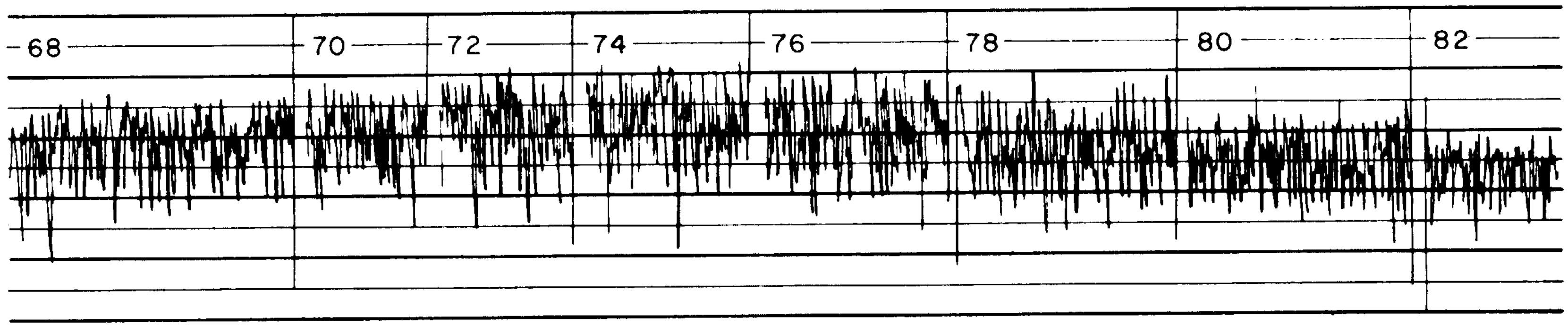
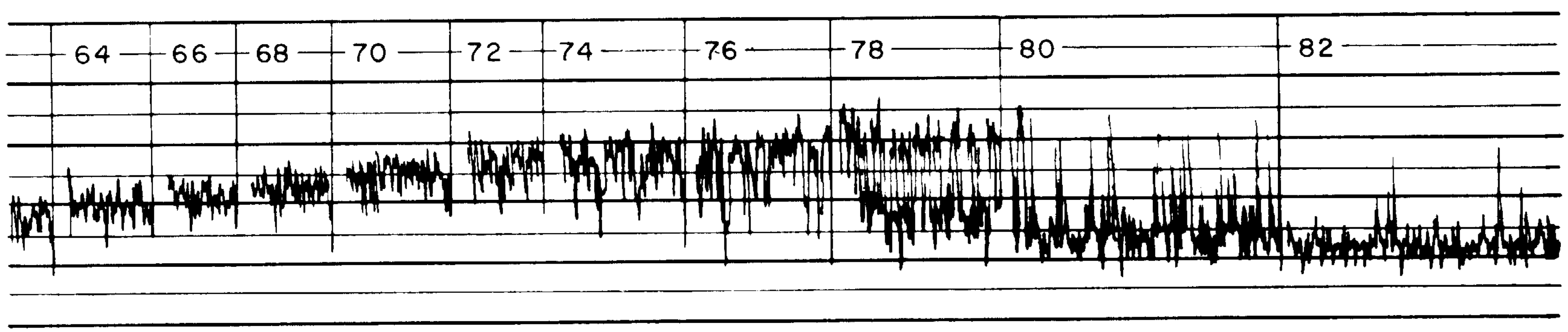


FIG. 4 DIMENSIONS AND LAYOUT OF HECKL'S EXPERIMENTAL PANEL (REF. 2)



a. LOW LEVEL EXCITATION



b. HIGH LEVEL EXCITATION

FIG. 5 RESPONSE ACCELERATION LEVELS FOR TWO VALUES OF RMS FORCE (REF. 2)

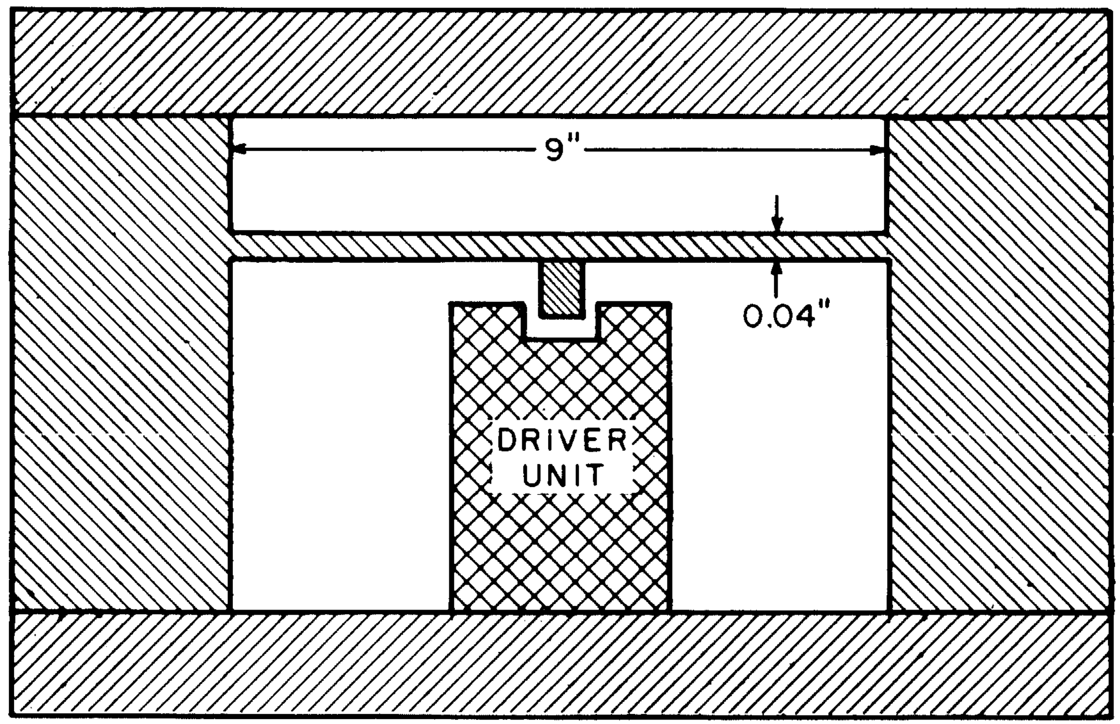


FIG. 6 DIAGRAM OF CLAMPED-CLAMPED BAR USED BY SMITH, SMITS AND LAMBERT (REF. 3)

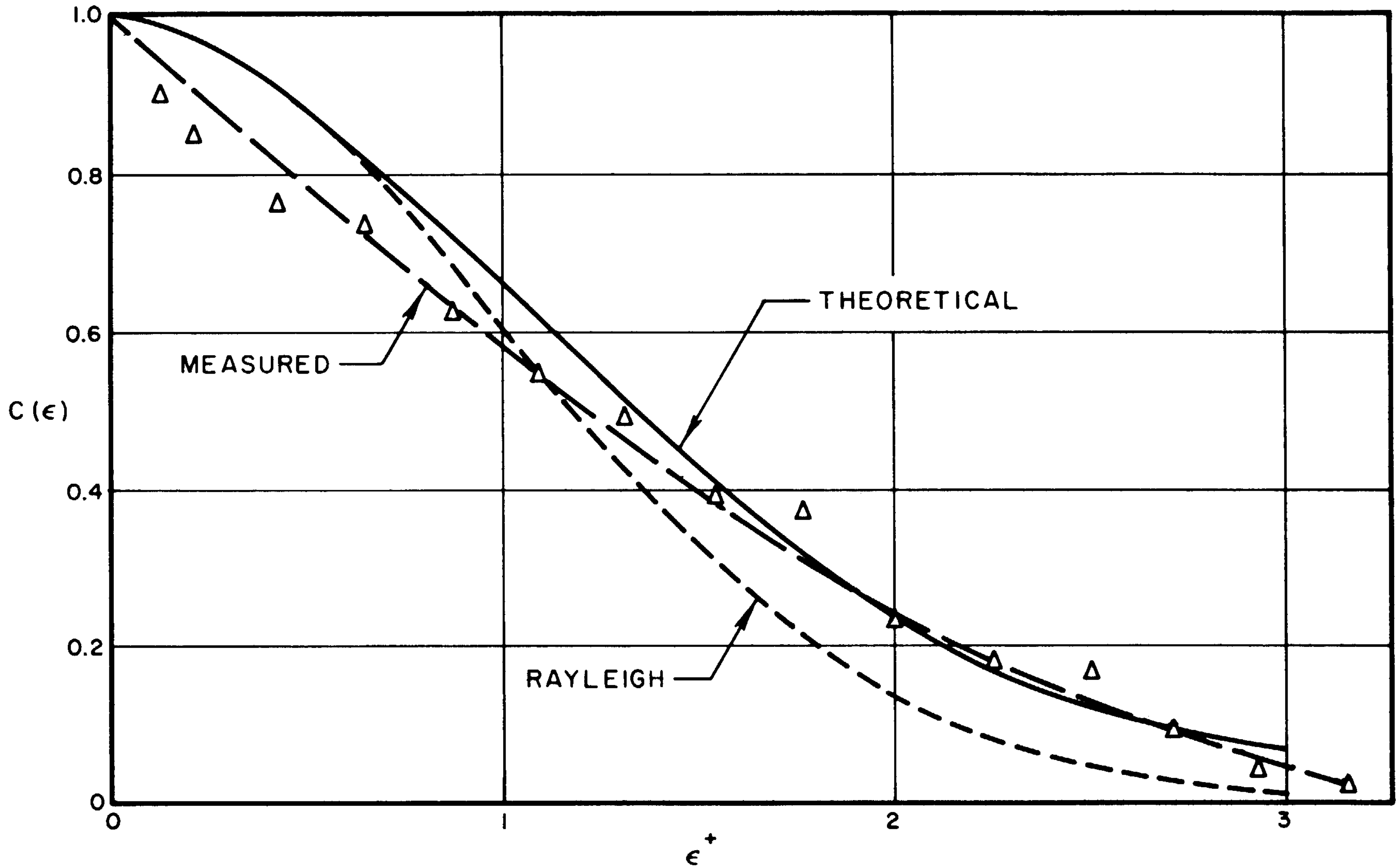


FIG. 7 POSITIVE CREST DISTRIBUTION
(REF. 3)

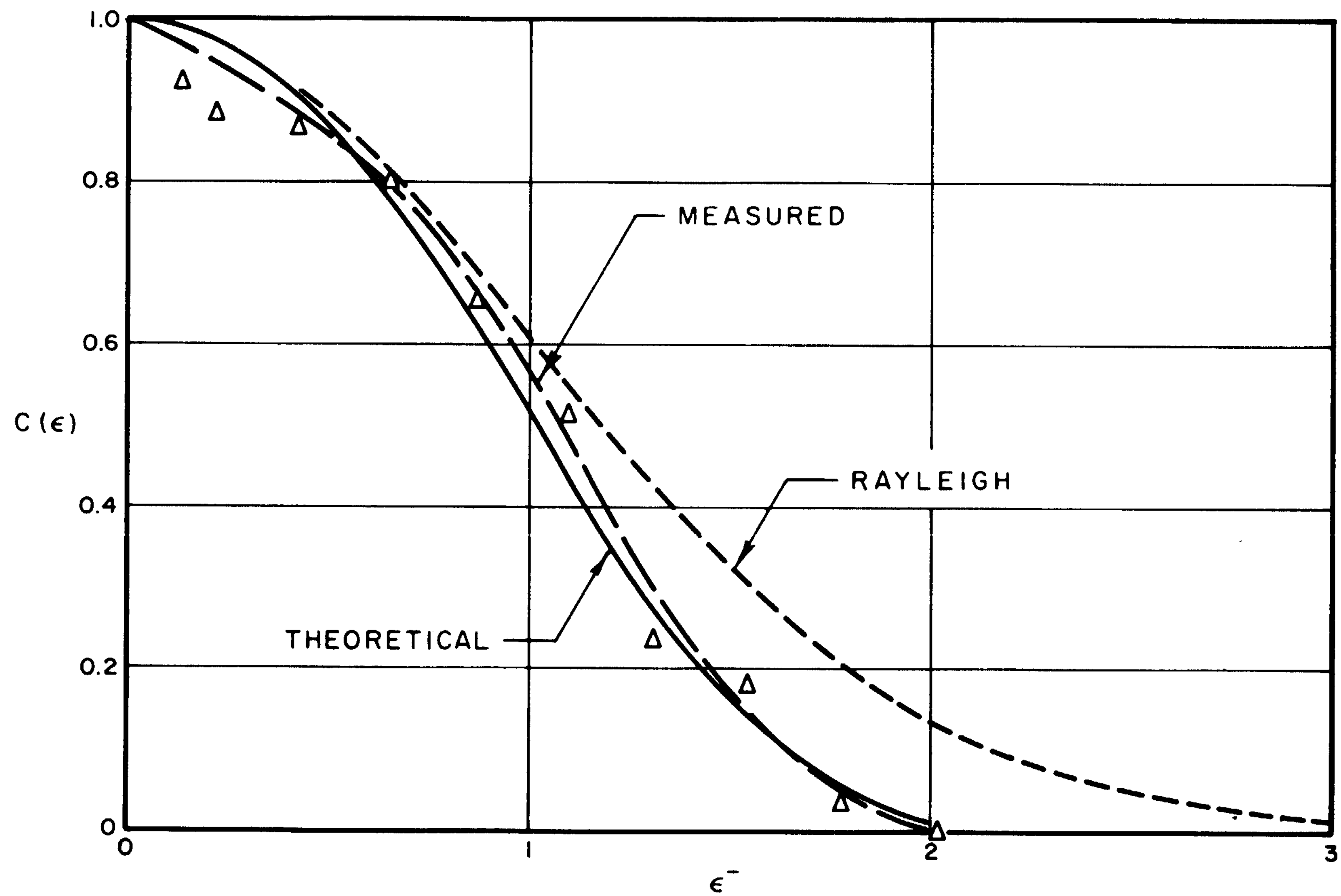
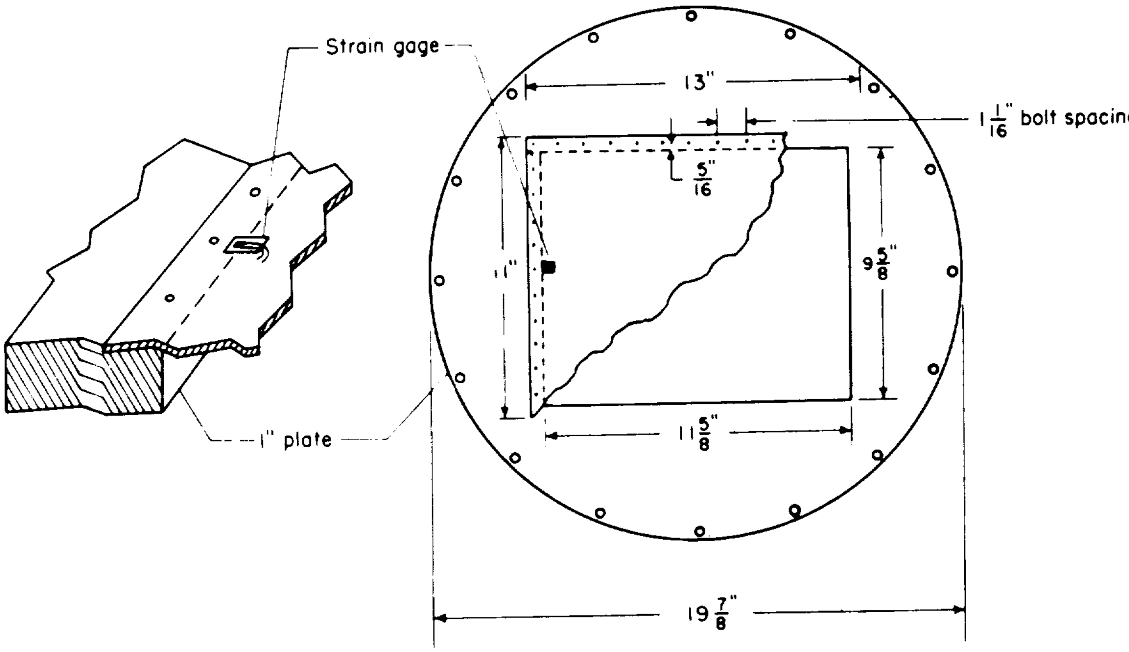
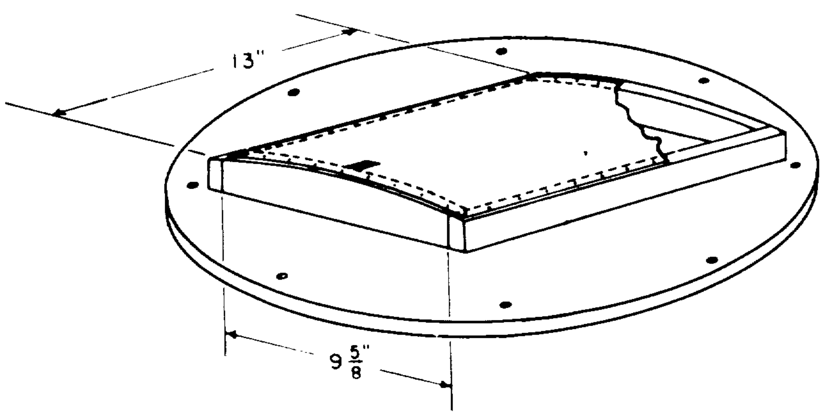


FIG. 8 NEGATIVE CREST DISTRIBUTION
(REF. 3)



a.) FLAT PANEL



b.) CURVED PANEL

FIG. 9 DETAILS OF TEST PANELS (REF. 4)

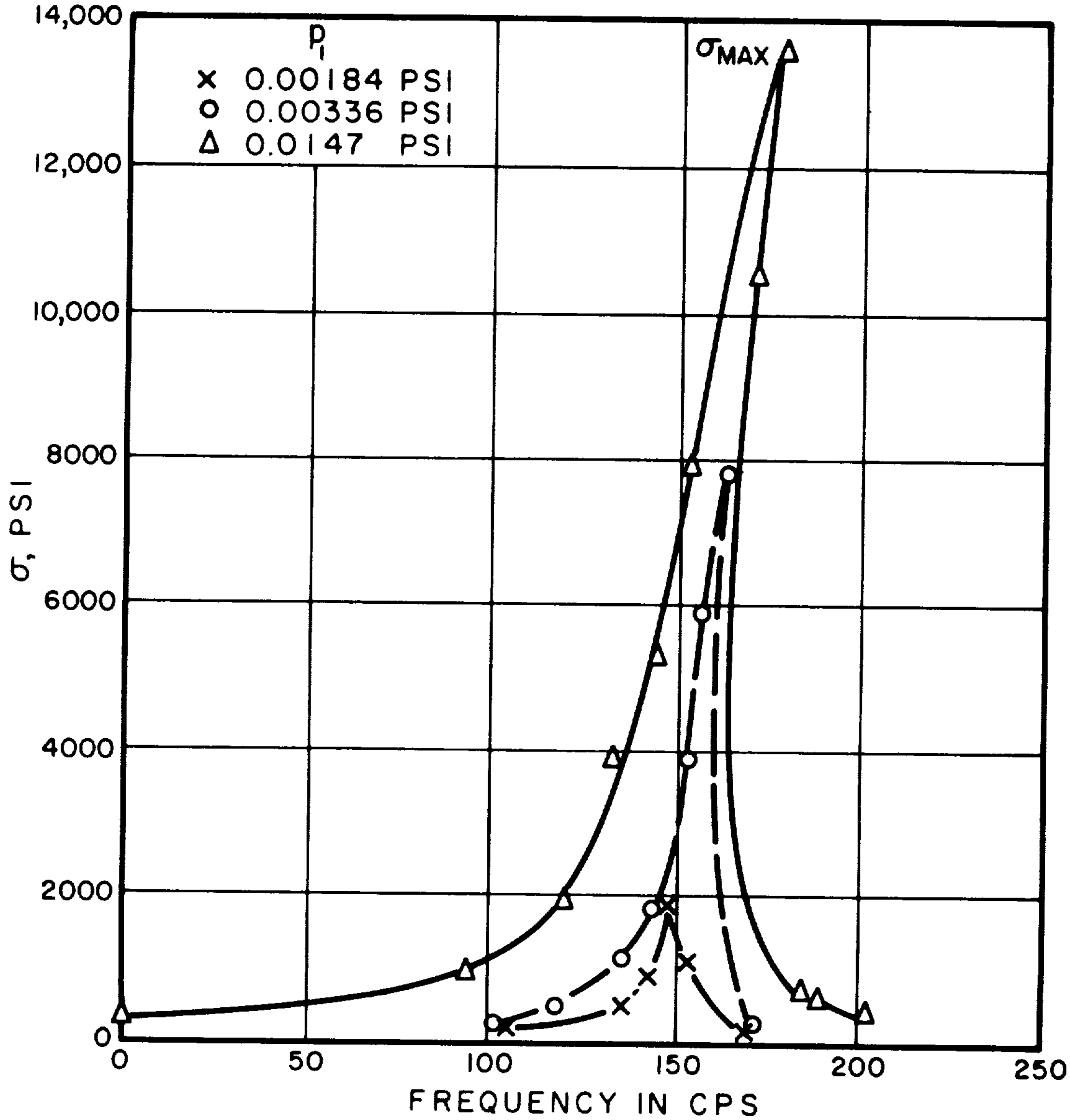


FIG. 10 FREQUENCY RESPONSE CHARACTERISTIC OF 40 MIL FLAT PANEL (REF. 4)

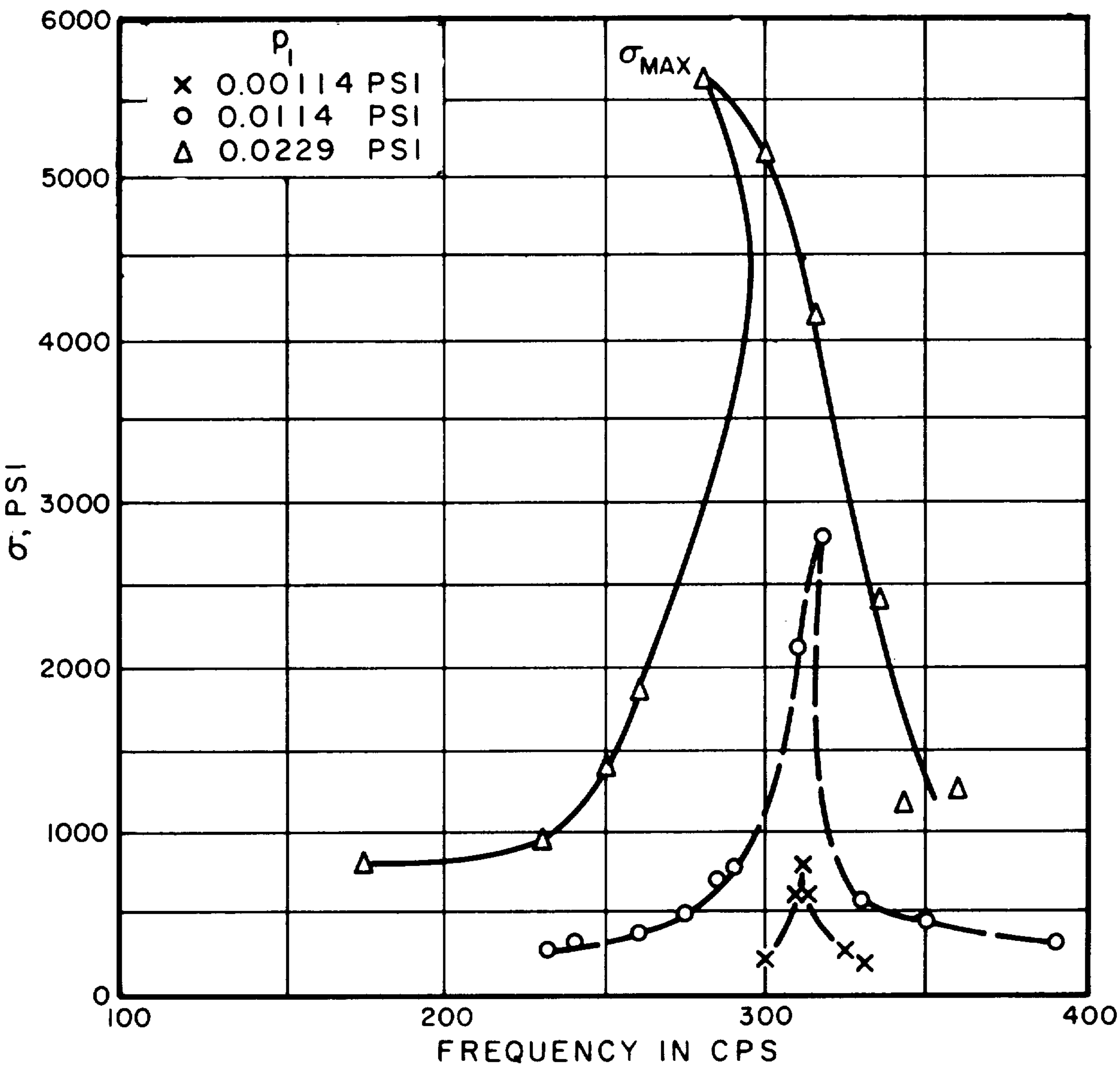


FIG. 11 FREQUENCY RESPONSE CHARACTERISTIC OF 32 MIL CURVED PANEL, RADIUS OF CURVATURE = 4 FT (REF. 4)

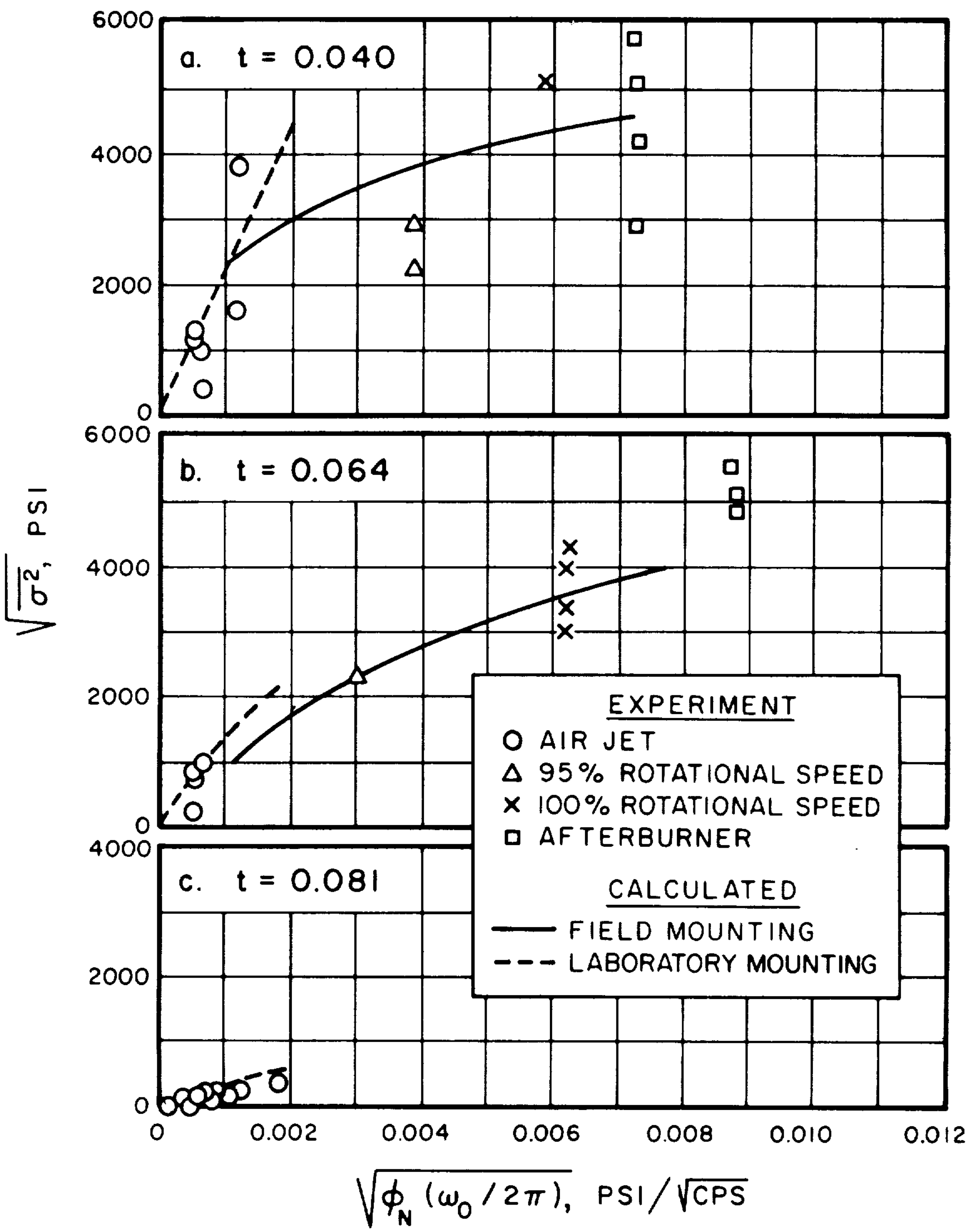


FIG. 12 COMPARISON OF CALCULATED AND MEASURED STRESS FOR FLAT PANELS (REF. 4).

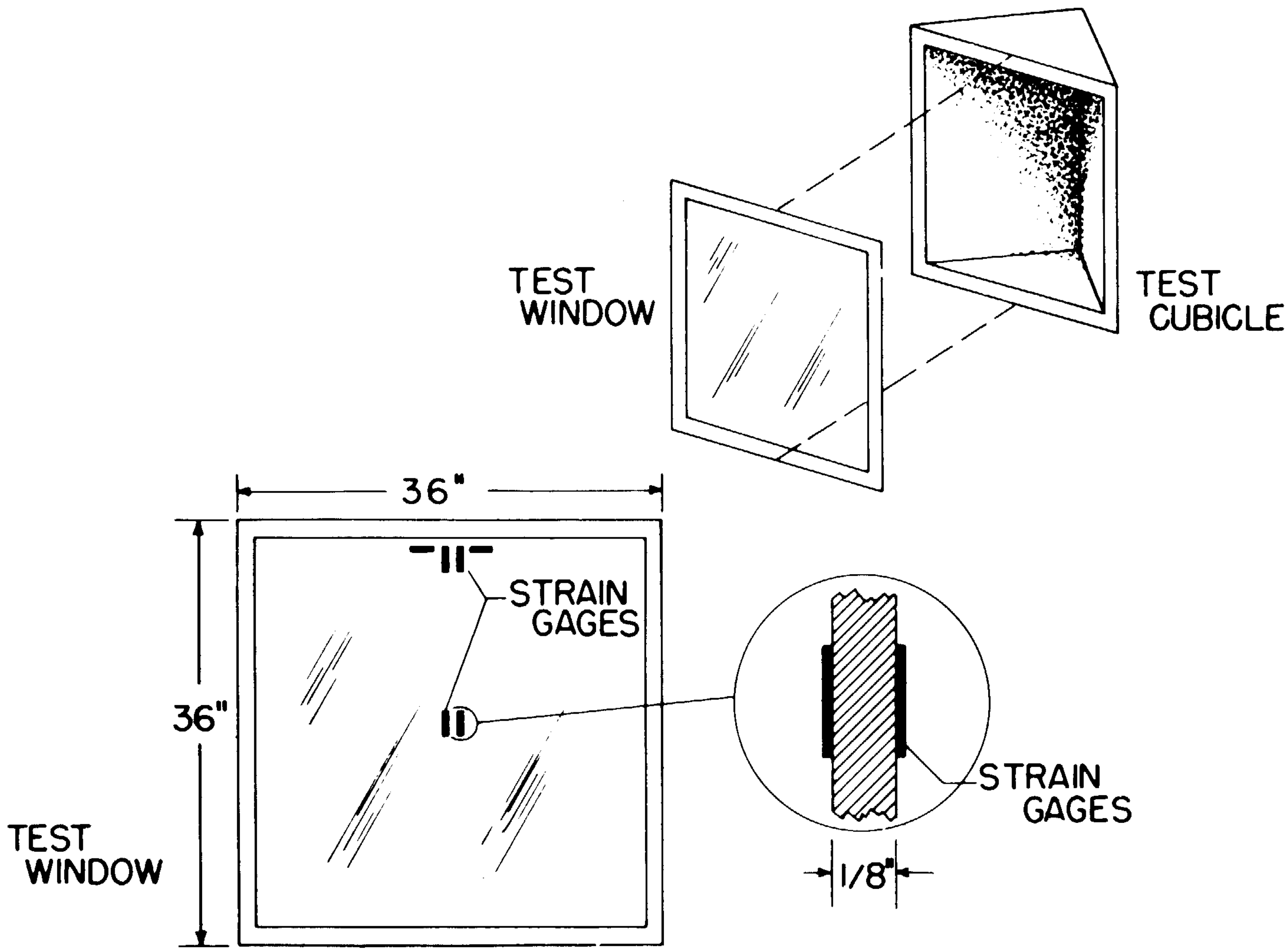


FIG.13 TEST WINDOW CONFIGURATION AND LOCATIONS OF STRAIN GAGES (REF. 5)

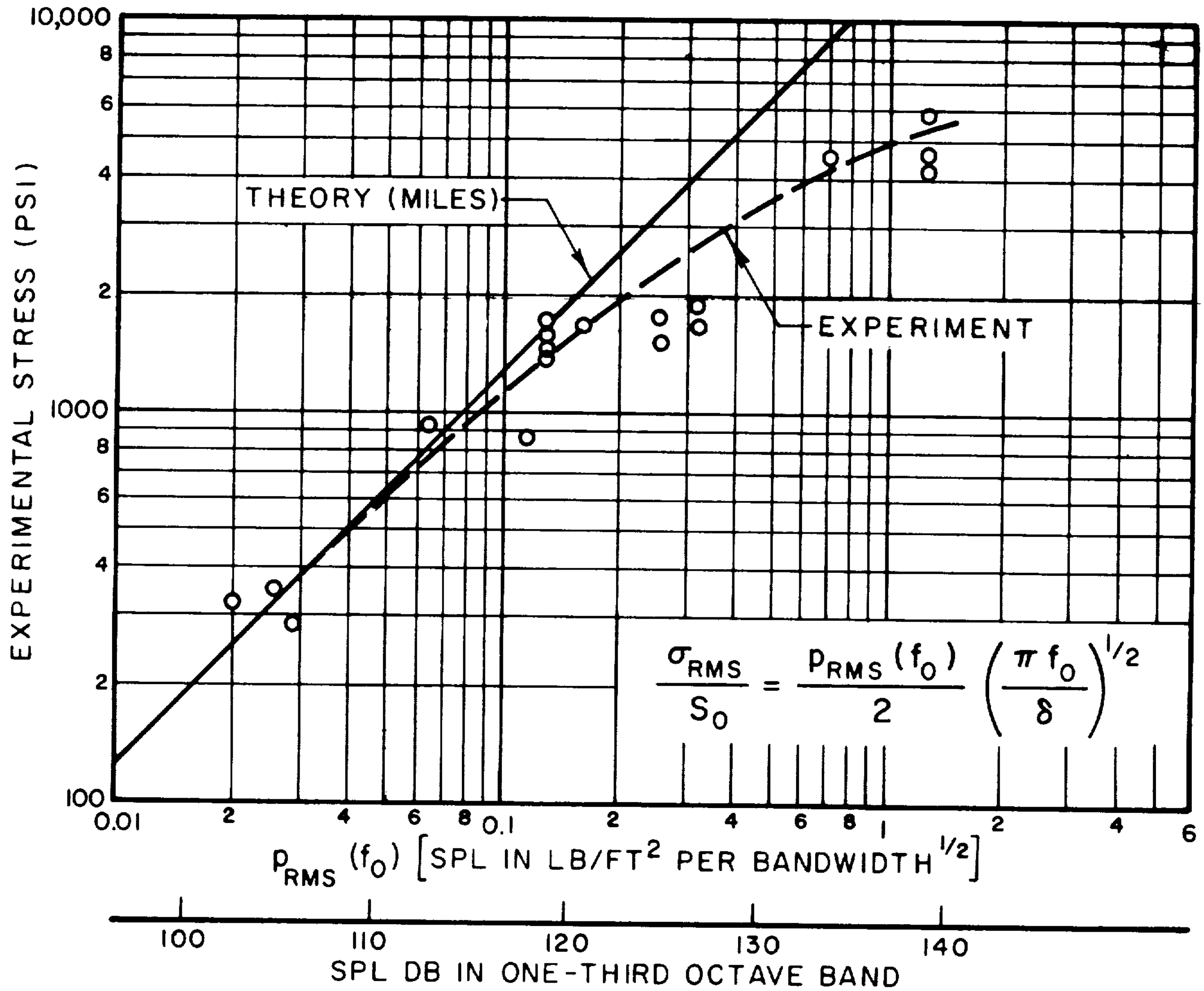


FIG. 14 COMPARISON OF EXPERIMENTAL AND THEORETICAL PEAK TENSILE STRESSES AT CENTER OF WINDOW (REF. 5)

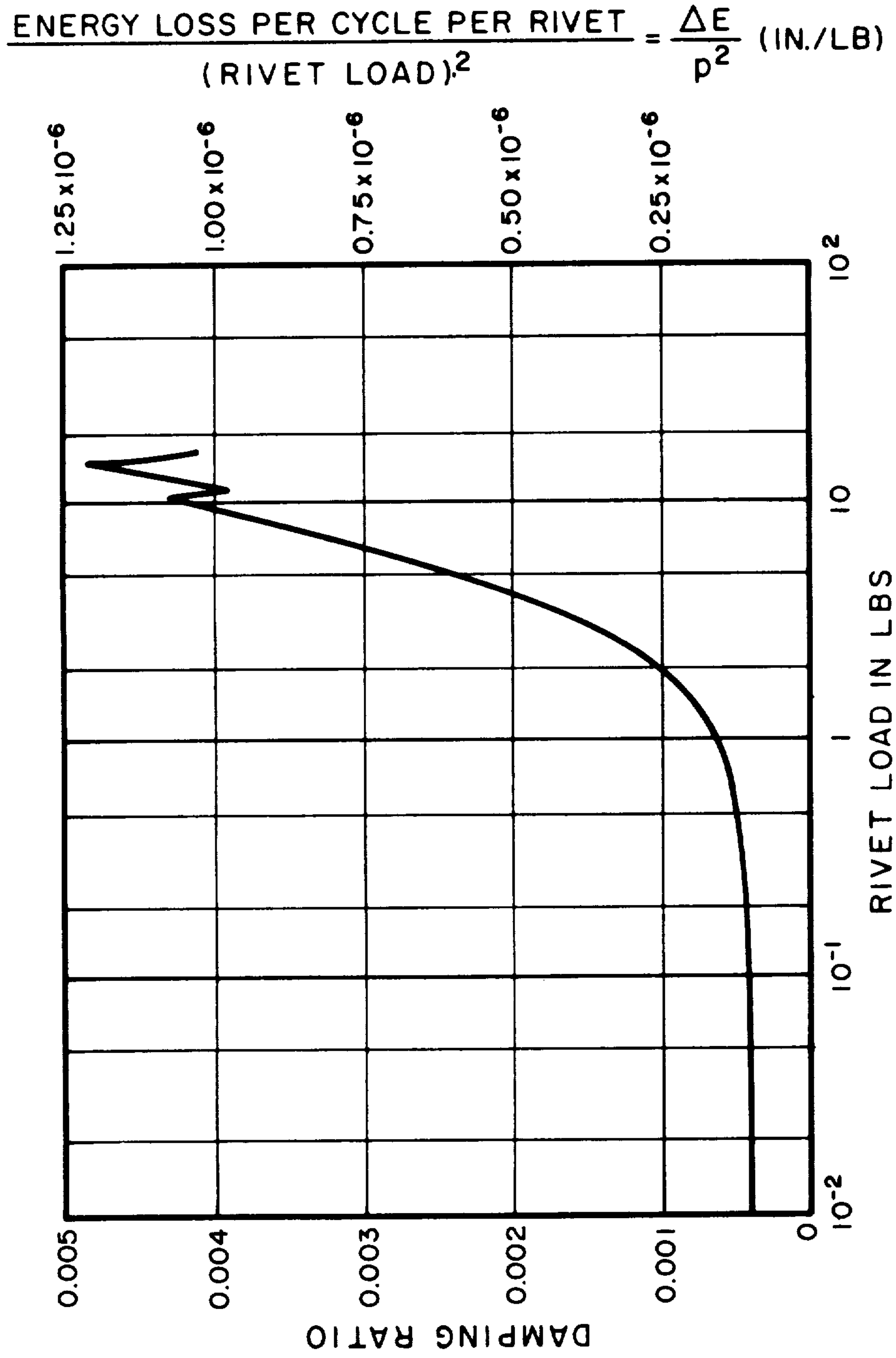


FIG. 15 VARIATION OF DAMPING OF A JOINTED BEAM WITH RIVET LOAD AMPLITUDE (REF. 8)